Estudando com o MATLAB

Curso de Extensão

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Aula do Dia!

Módulos		Conteúdos Teóricos
3º & 4º 23/09 & 24/09	- Aplicações Partes I e II	- Parte I - Matemática; - Física; - Engenharias; - Parte II; - Estatística; - Processamento de Sinais; - Processamento de Imagens.

Symbolic Math Toolbox - SMT

Calculus	Differentiation, integration, limits, summation, and Taylor series
Simplifications and Substitutions	Methods of simplifying algebraic expressions
Variable-Precision Arithmetic	Numerical evaluation of mathematical expressions to any specified accuracy
Linear Algebra	Inverses, determinants, eigenvalues, singular value decomposition, and canonical forms of symbolic matrices
Solving Equations	Symbolic and numerical solutions to algebraic and differential equations
Special Mathematical Functions	Special functions of classical applied mathematics
Using Maple® Functions	How to use the maple command to access Maple® functions directly

A Derivada SMT

```
>> %· Cadeia de caracteres

diff ('3*x^4+4*x')

ans =

12*x^3+4


>> %· Objeto 'symbol' definido pelas funções syms e sym

syms x % define o símbolo x

y=3*x^4+4*x;

diff (y)

ans =

12*x^3+4
```

A Integral SMT

```
>> %· Cadeia de caracteres

int ('3*x^4+4*x')

ans =

3/5*x^5+2*x^2


>> %· Objeto 'symbol' definido pelas funções syms e sym

syms x % define o símbolo x

y=3*x^4+4*x;

int (y)

ans =

3/5*x^5+2*x^2
```

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Implemente Agora!!!

Matriz SMT

```
>> n = 3;
syms x
B = x.^((0:n)'*(0:n))

B =

[   1,   1,   1,   1]
[   1,   x, x^2, x^3]
[   1, x^2, x^4, x^6]
[   1, x^3, x^6, x^9]
```

Manipulando com SMT

Manipulando Polinômios: expand(S) e factor(S)

```
>> syms x
>> f=x^3+x^2+10;
>> h=x^4+(x-3)^3+10;
>> w=f*h;
>> expand(w)

ans =

x^7+2*x^6-8*x^5+28*x^4+20*x^3-107*x^2+270*x-170



>> factor(w)

ans =

(x^3+x^2+10)*(x^4+x^3-9*x^2+27*x-17)
```

Manipulando com SMT

Expressões e Matrizes:

```
>> syms x
>> g=10*sin(x)+x^2;
>> int(g)

ans =

-10*cos(x)+1/3*x^3
```

```
>> syms x
>> mat=[4 5 x;2 x 2; 20 x x^2];
>> det(mat)

ans =

4*x^3-8*x-28*x^2+200
```

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Manipulando com SMT

Expressões e Matrizes:

```
>> syms s
H = (s^3 +2*s^2 +5*s +10)/(s^2 + 5);
simplify(H)
[N,D] = numden(H)

ans =

s+2

N =


s^3+2*s^2+5*s+10

D =


s^2+5
```

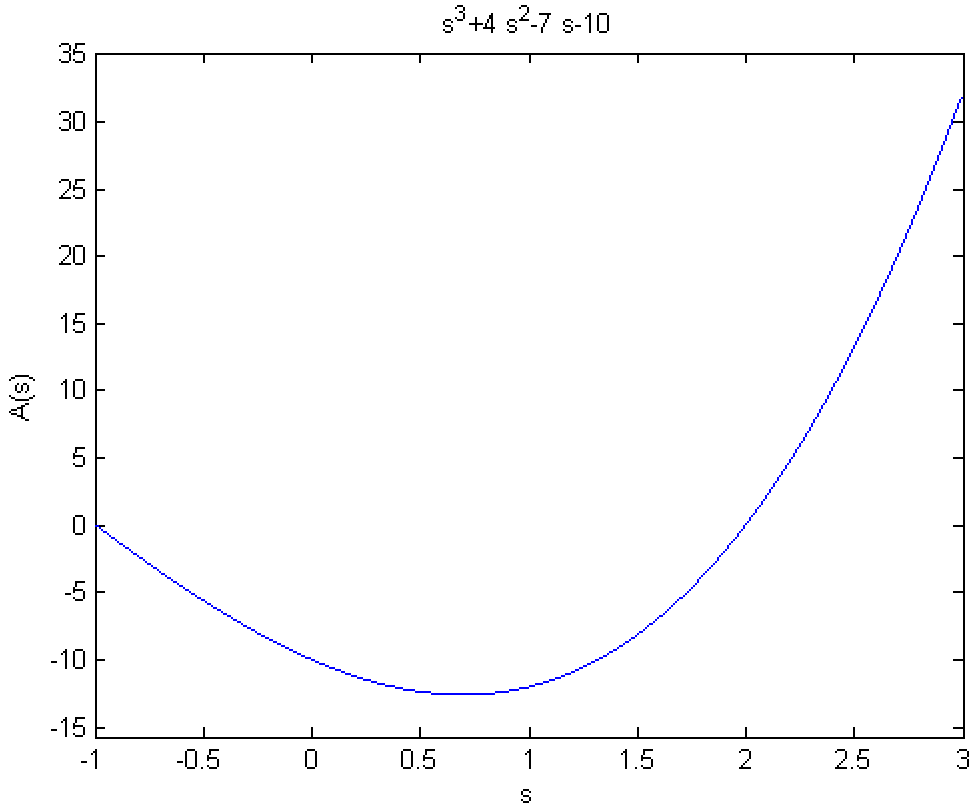
Resolvendo Sistemas com SMT

```
>> syms x y
% Encontrar as intersecções entre uma
% circunferência x2+y2=1 centrada em (0,0)
% de raio 1 com a reta x+y=0
k=solve('x^2 + y^2=1', 'x+y=0')

k =

    x: [2x1 sym]
    y: [2x1 sym]

>> pretty(k.x)

                                  [         1/2]
                                  [- 1/2 2     ]
                                  [            ]
                                  [        1/2 ]
                                  [ 1/2 2      ]
>> pretty(k.y)

                                  [        1/2 ]
                                  [ 1/2 2      ]
                                  [            ]
                                  [         1/2]
                                  [- 1/2 2     ]
```

Gráficos no SMT

$A(s) = s^3 + 4s^2 - 7s - 10, \; [-1, 3]$:

```
syms s
a = [1 4 -7 -10];
A = poly2sym(a,s)
ezplot(A,-1,3), ylabel('A(s)')
```



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Implemente Agora!!!

Resumo SMT

```
>> %DIFERENCIAÇÃO
>> diff('x^2+y*x','x')

ans =

2*x+y


>> diff('x^2+y*x','y')

ans =

x


>>
```

```
>> %INTEGRAÇÃO
>> int('x^2*cos(x)','x')

ans =

x^2*sin(x)-2*sin(x)+2*x*cos(x)


>> int('x^2*cos(x)',0,1)

ans =

-sin(1)+2*cos(1)


>>
```

```
>> %LIMITES
>> syms x;limit(sin(x)/x,x,0,'left')

ans =

1


>>
```

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Implemente Agora!!!

Resumo SMT

```
>> %SOMATORIOS
>> syms k n; symsum(1/k - 1/(k+1),1,n)

ans =

-1/(n+1)+1


>> syms k n; symsum(1/k - 1/(k+1),1,inf)

ans =

1

>>
```

```
>> %SISTEMAS LINEARES OU NÃO LINEARES
>> [x, y] = solve('x^2-y=2','y-2*x=5')

x =

 1+2*2^(1/2)
 1-2*2^(1/2)


y =

 7+4*2^(1/2)
 7-4*2^(1/2)
```

```
>> %POLINOMIO DE TAYLOR
>> syms x; taylor(cos(x),x,10)

ans =

1-1/2*x^2+1/24*x^4-1/720*x^6+1/40320*x^8

>>
```

Resolvendo Problemas!

The problem-solving process for a computational problem can be outlined as follows:

1. Define the problem.
2. Create a mathematical model.
3. Develop a computational method for solving the problem.
4. Implement the computational method.
5. Test and assess the solution.

Exemplo I - Cinemática

A small object is launched into flight from the ground at a speed of 50 miles/hour at 30 degrees above the horizontal over level ground. Determine the time of flight and the distance traveled when the ball returns to the ground.

1. Define the problem.
2. Create a mathematical model.
3. Develop a computational method for solving the problem.
4. Implement the computational method.
5. Test and assess the solution.

Exemplo I - Cinemática

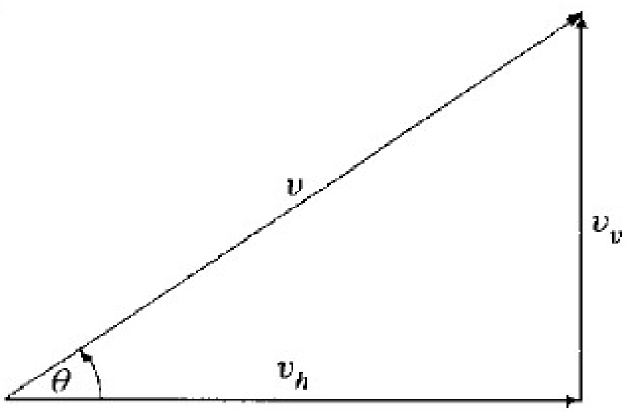
2. **Mathematical Model:**

To pose this problem in terms of a mathematical model, we first need to define the notation:

- Time: t (s), with $t = 0$ when the object is launched.
- Initial velocity magnitude: $v = 50$ miles/hour.
- Initial angle: $\theta = 30°$.
- Horizontal position of ball: $x(t)$ (ft).
- Vertical position of ball: $y(t)$ (ft).
- Acceleration of gravity: $g = 32.2$ ft/s^2, directed in negative y direction.

From basic trigonometry, we know th '

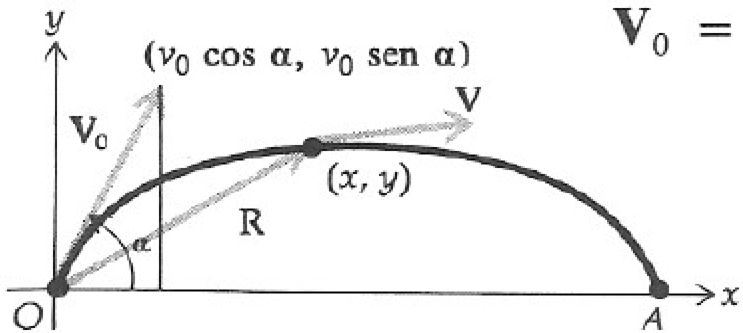
$$v_h = v\cos\theta$$
$$v_v = v\sin\theta$$



$$x(t) = vt\cos\theta$$

$$y(t) = vt\sin\theta - \frac{1}{2}gt^2$$

Exemplo I - Cinemática



$$\mathbf{V}_0 = v_0 \cos\alpha\mathbf{i} + v_0 \operatorname{sen}\alpha\mathbf{j}$$

$$\mathbf{R}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$$

$$\mathbf{V}(t) = \mathbf{R}'(t)$$

$$\mathbf{A}(t) = \mathbf{V}'(t)$$

$\mathbf{F} = -mg\mathbf{j} \rightarrow \mathbf{F} = m\mathbf{A} \rightarrow m\mathbf{A} = -mg\mathbf{j} \rightarrow \mathbf{A} = -g\mathbf{j}$ A segunda lei de Newton

Como $\mathbf{A}(t) = \mathbf{V}'(t)$, $\mathbf{V}'(t) = -g\mathbf{j} \rightarrow \mathbf{V}(t) = -gt\mathbf{j} + \mathbf{C}_1$

Quando $t = 0$, $\mathbf{V} = \mathbf{V}_0$. Assim $\mathbf{C}_1 = \mathbf{V}_0$. Logo, $\mathbf{V}(t) = -gt\mathbf{j} + \mathbf{V}_0 = \mathbf{R}'(t)$

Integrando $\mathbf{R}(t) = -\frac{1}{2}gt^2\mathbf{j} + \mathbf{V}_0 t + \mathbf{C}_2$ (2)

Quando $t = 0$, $\mathbf{R} = 0$. Logo, $\mathbf{C}_2 = 0 \rightarrow \mathbf{R}(t) = -\frac{1}{2}gt^2\mathbf{j} + \mathbf{V}_0 t$

Substituindo o valor de $\mathbf{V}_0$ de (2) no resultado acima, obtemos

$$\mathbf{R}(t) = -\frac{1}{2}gt^2\mathbf{j} + (v_0 \cos\alpha\mathbf{i} + v_0 \operatorname{sen}\alpha\mathbf{j})t = tv_0 \cos\alpha\mathbf{i} + (tv_0 \operatorname{sen}\alpha - \frac{1}{2}gt^2)\mathbf{j}$$

Exemplo I - Cinemática

3. **Computational Method:**

   Using the model developed above, expressions for the desired results can be obtained. The object will hit the ground when its vertical position is zero

   $$y(t) = vt\sin\theta - \frac{1}{2}gt^2 = 0$$

   which can be solved to yield two values of time

   $$t = 0, \quad \frac{2v\sin\theta}{g}$$

   The second of the two solutions indicates that the object will return to the ground at the time

   $$t_g = \frac{2v\sin\theta}{g}$$

   The horizontal position (distance of travel) at this time is

   $$x(t_g) = vt_g\cos\theta$$

Exemplo I - Cinemática

4. Computational Implementation:

```
% Valores iniciais
g = 32.2; % gravidade, ft/s^2
v = 50 * 5280/3600; % velocidade inicial, ft/s
theta = 30 * pi/180; % angulo inicial, radians
tg = 2 * v * sin(theta)/g % Tempo de retorno, s
xg = v * cos(theta) * tg % Distancia percorrida
% Gráfico
t = linspace(0,tg,256);
x = v * cos(theta) * t;
y = v * sin(theta) * t - g/2 * t.^2;
plot(x,y), axis([ 0 xg 0 max(y) ]), grid,
xlabel('Distancia (ft)'), ylabel('Altura (ft)')
title('Trajetoria')
```

Exemplo I - Cinemática

A small object is launched into flight from the ground at a speed of 50 miles/hour at 30 degrees above the horizontal over level ground. Determine the time of flight and the distance traveled when the ball returns to the ground.

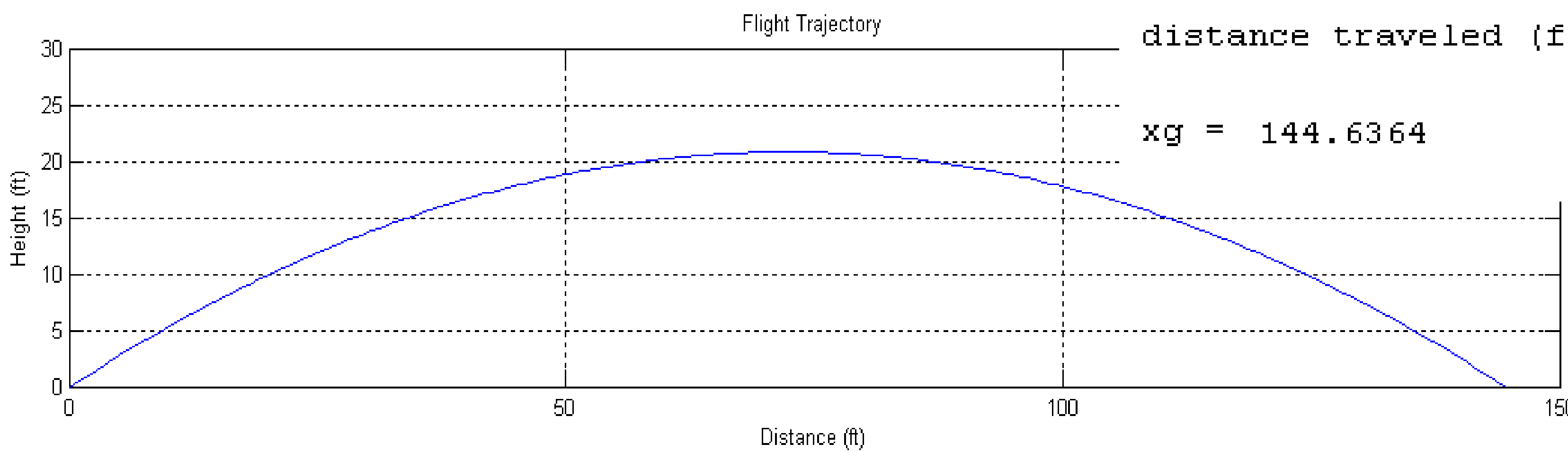
5. Testing and Assessing the Solution:

```
time of flight (s):

tg =  2.2774

distance traveled (ft):

xg =  144.6364
```



Exemplo II - Cinemática

Considerando o problema anterior, determine as posições x(t) e y(t) em qualquer tempo??? Esbouce o gráfico!

Exemplo III – Resposta em Freq.

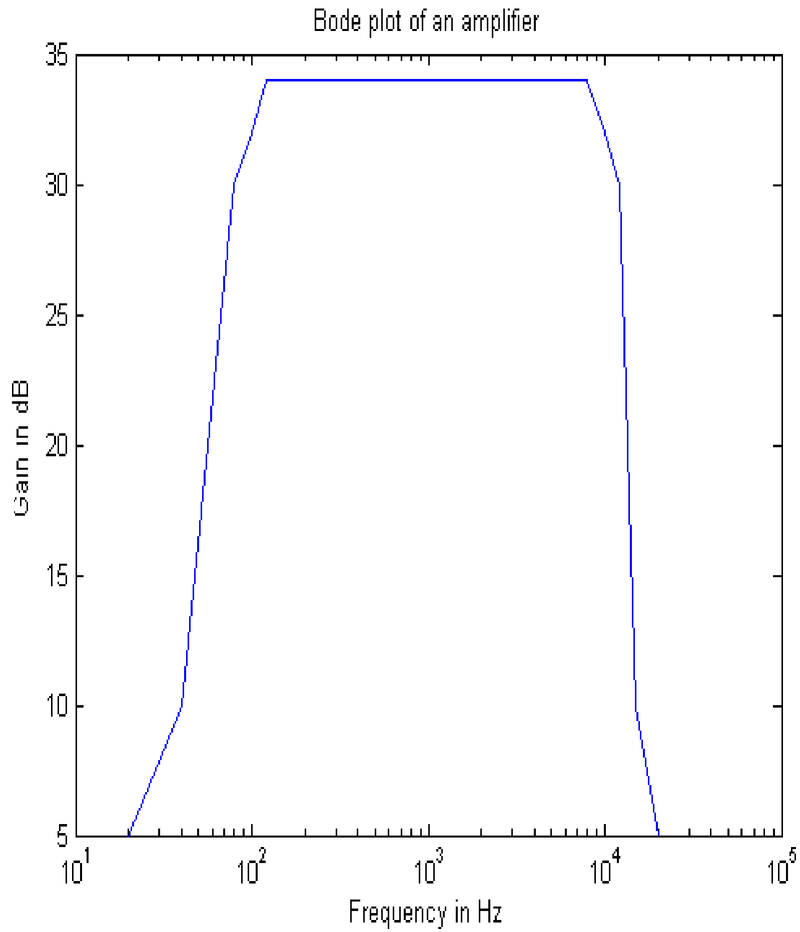
The gain versus frequency of a capacitively coupled amplifier is shown below. Draw a graph of gain versus frequency using a logarithmic scale for the frequency and a linear scale for the gain.

Frequency (Hz)	Gain (dB)	Frequency (Hz)	Gain (dB)
20	5	2000	34
40	10	5000	34
80	30	8000	34
100	32	10000	32
120	34	12000	30

Exemplo III – Resposta em Freq.

Frequency (Hz)	Gain (dB)	Frequency (Hz)	Gain (dB)
20	5	2000	34
40	10	5000	34
80	30	8000	34
100	32	10000	32
120	34	12000	30

```
>> % Bode plot for capacitively coupled amplifier
f = [20 40 80 100 120 2000 5000 8000 10000 ...
12000 15000 20000];
g = [ 5 10 30 32 34 34 34 34 32 30 10 5];
semilogx(f, g)
title('Bode plot of an amplifier')
xlabel('Frequency in Hz')
ylabel('Gain in dB')
>>
```



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Implemente Agora!!!

Exemplo IV – Conversor A/D

A 3-bit A/D converter, with an analog input x and digital output y, is represented by the equation:

$$
\begin{aligned}
y &= 0 & x &< -2.5 \\
&= 1 & -2.5 \leq x &< -1.5 \\
&= 2 & -1.5 \leq x &< -0.5 \\
&= 3 & -0.5 \leq x &< 0.5 \\
&= 4 & 0.5 \leq x &< 1.5 \\
&= 5 & 1.5 \leq x &< 2.5 \\
&= 6 & 2.5 \leq x &< 3.5 \\
&= 7 & x &\geq 3.5
\end{aligned}
$$

Write a MATLAB program to convert analog signal x to digital signal y. Test the program by using an analog signal with the following amplitudes: -1.25, 2.57 and 6.0.

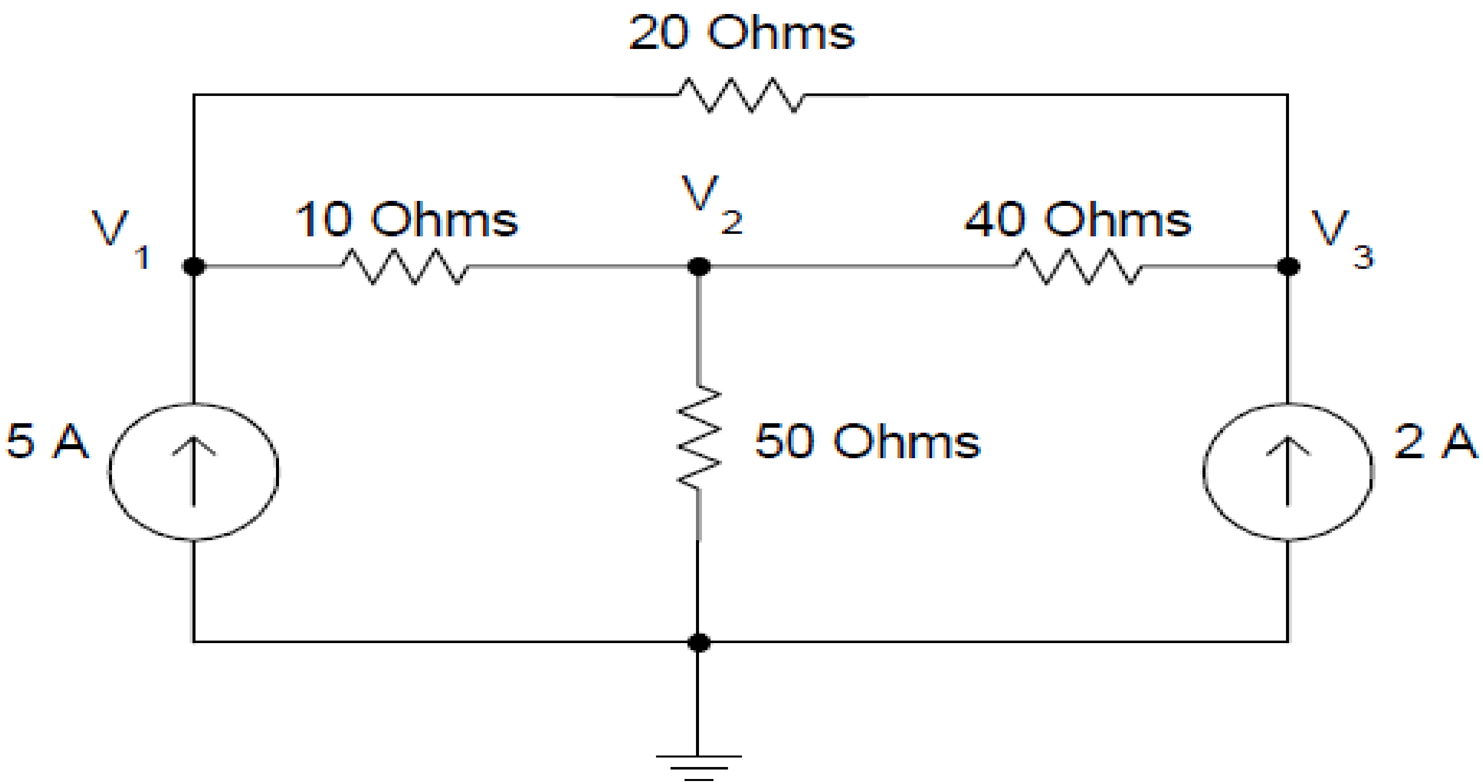
Exemplo IV – Conversor A/D

```
function Y_dig = bitatd_3(X_analog)
%
% bitatd_3 is a function program for obtaining
% the digital value given an input analog
% signal
%
% usage: Y_dig = bitatd_3(X_analog)
% Y_dig is the digital number (in integer form)
% X_analog is the analog input (in decimal form)
%
if X_analog < -2.5
Y_dig = 0;
elseif X_analog >= -2.5 & X_analog < -1.5
Y_dig = 1;
elseif X_analog >= -1.5 & X_analog < -0.5
Y_dig = 2;
elseif X_analog >= -0.5 & X_analog < 0.5
Y_dig = 3;
elseif X_analog >= 0.5 & X_analog < 1.5
Y_dig = 4;
elseif X_analog >= 1.5 & X_analog < 2.5
Y_dig = 5;
elseif X_analog >= 2.5 & X_analog < 3.5
Y_dig = 6;
else
Y_dig = 7;
end
Y_dig;
end
```

```
>> y1 = bitatd_3(-1.25)
y2 = bitatd_3(2.57)
y3 = bitatd_3(6.0)

y1 =

     2


y2 =

     6


y3 =

     7

>>
```

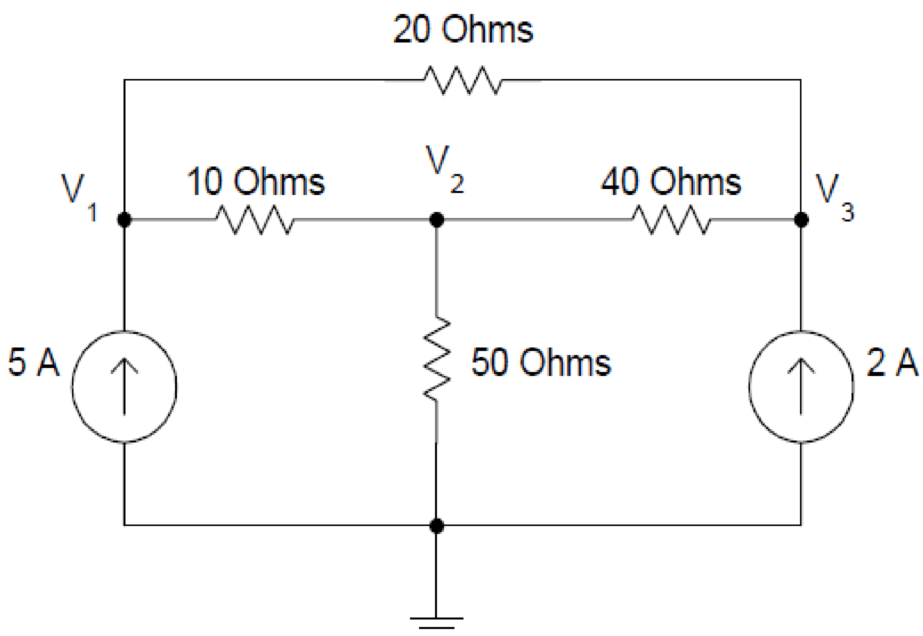
Exemplo V – Circuitos Elétricos

For the circuit shown below, find the nodal voltages V_1, V_2 and V_3.



Exemplo V – Circuitos Elétricos

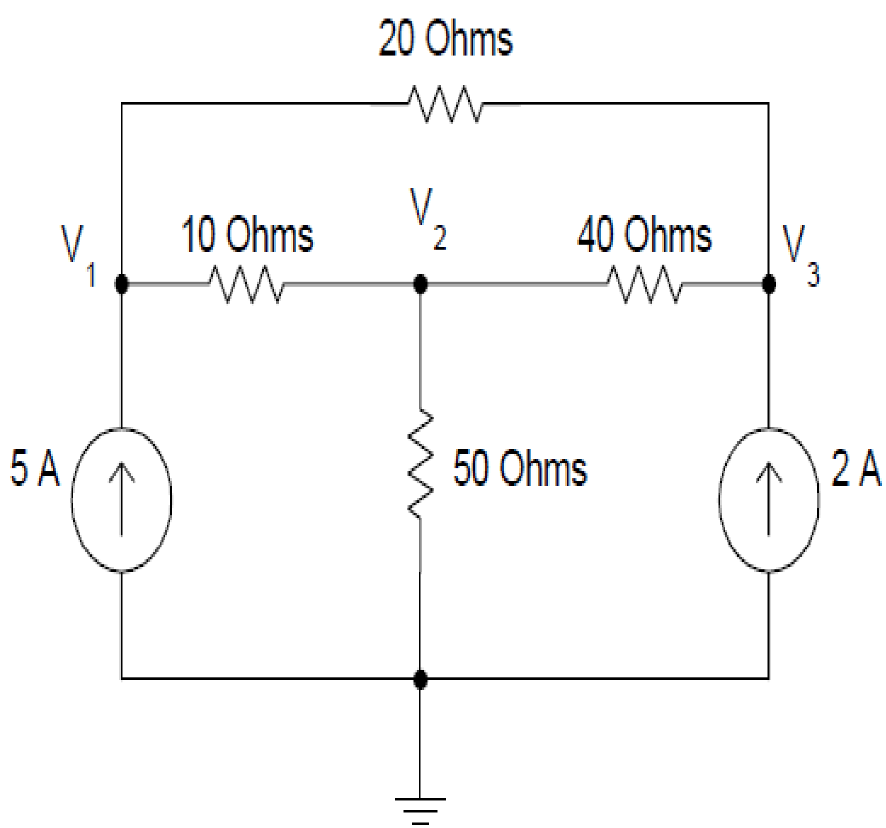
For the circuit shown below, find the nodal voltages V_1, V_2 and V_3.



$$\begin{bmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.145 & -0.025 \\ -0.05 & -0.025 & 0.075 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

Exemplo V – Circuitos Elétricos

For the circuit shown below, find the nodal voltages V_1, V_2 and V_3.




```
>> %Matriz Y
Y = [ 0.15 -0.1 -0.05;
     -0.1 0.145 -0.025;
     -0.05 -0.025 0.075];
%Vetor I
I = [5;0;2];
%Operação V=Inversa(Y)*I
v = inv(Y)*I
v =
   404.2857
   350.0000
   412.8571
```


Implemente Agora!!!

Exemplo VI – Circuitos Elétricos

Assume that for Figure 5.2 C = 10 μF, use MATLAB to plot the voltage across the capacitor if R is equal to (a) 1.0 kΩ, (b) 10 kΩ and (c) 0.1 kΩ.

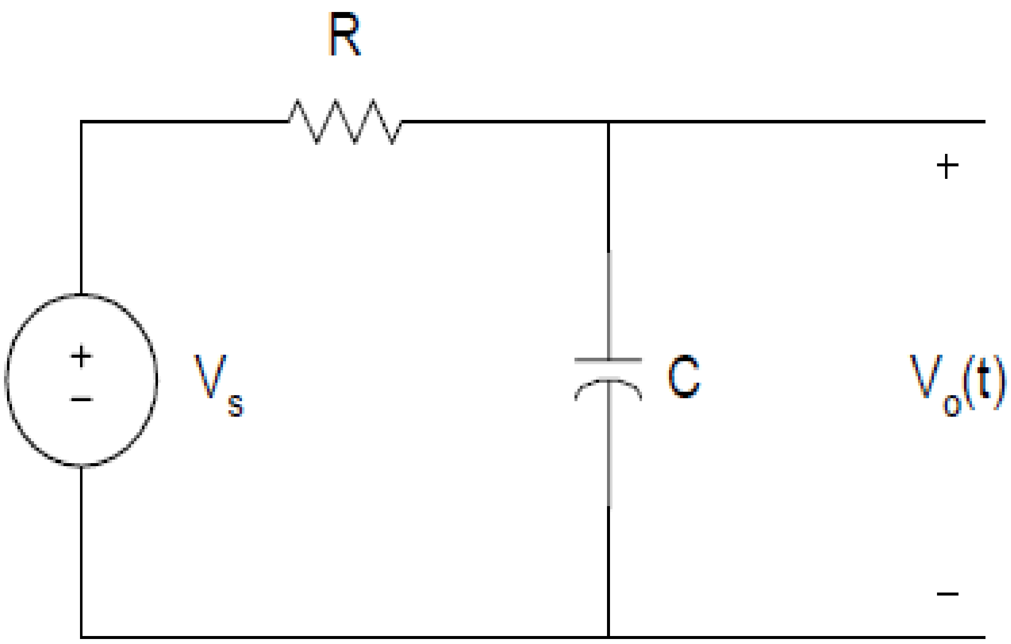


Figure 5.2 Charging of a Capacitor

Exemplo VI – Circuitos Elétricos

Assume that for Figure 5.2 C = 10 μF, use MATLAB to plot the voltage across the capacitor if R is equal to (a) 1.0 kΩ, (b) 10 kΩ and (c) 0.1 kΩ.

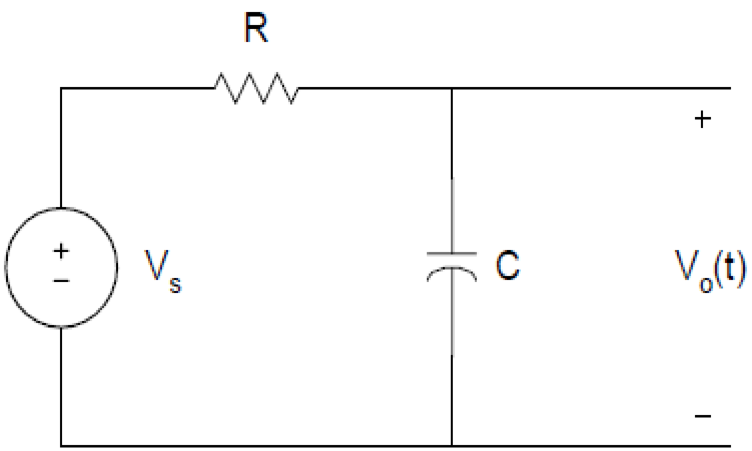


Figure 5.2 Charging of a Capacitor

Using KCL, we get

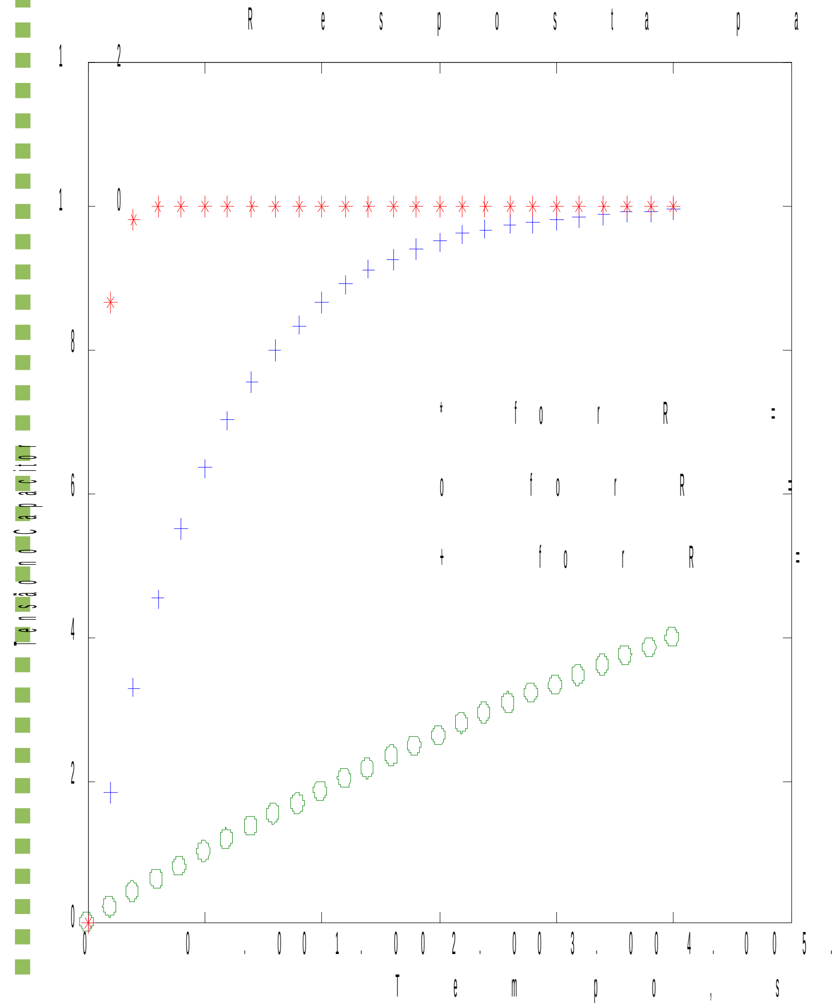
$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0$$

If the capacitor is initially uncharged, that is $v_0(t) = 0$ at $t = 0$, the solution to Equation (5.3) is given as

$$v_0(t) = V_S\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)$$

Exemplo VI – Circuitos Elétricos

```
t = 0:0.002:0.05; %Eixo-Tempo
c = 10e-6; %Capacitor
r1 = 1e3;tau1 = c*r1;%Primeira relação RC
v1 = 10*(1-exp(-t/tau1));
r2 = 10e3; tau2 = c*r2;%Segunda relação RC
v2 = 10*(1-exp(-t/tau2));
r3 = .1e3;tau3 = c*r3;%Terceira relação RC
v3 = 10*(1-exp(-t/tau3));
%Grafico
plot(t,v1,'+',t,v2,'o', t,v3,'*')
axis([0 0.06 0 12])
title('Resposta para três constantes de tempo')
xlabel('Tempo, s');ylabel('Tensão no Capacitor')
text(0.03, 5.0, '+ for R = 1 Kilohms')
text(0.03, 6.0, 'o for R = 10 Kilohms')
text(0.03, 7.0, '* for R = 0.1 Kilohms')
```



Exemplo VII – Circuitos Elétricos

For Figure 5.2, $V_S = 10\text{V}$, $R = 10{,}000\ \Omega$, $C = 10\mu\text{F}$. Find the output voltage $v_0(t)$, between the interval 0 to 20 ms, assuming $v_0(0) = 0$ and by (a) using a numerical solution to the differential equation; and (b) analytical solution.

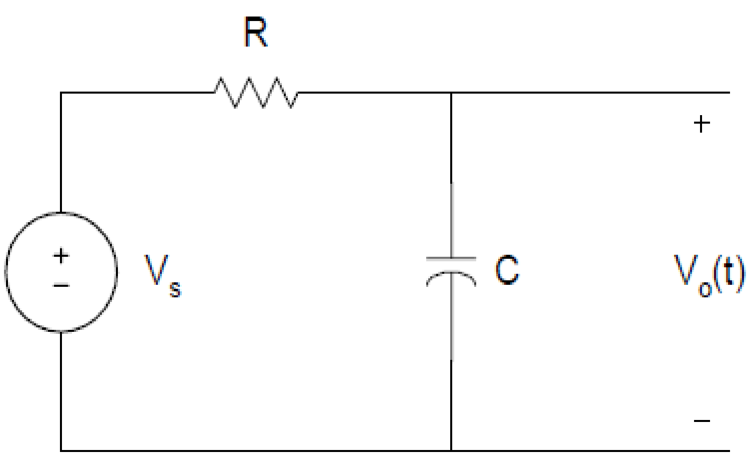


Figure 5.2 Charging of a Capacitor

Solution

From Equation (5.3), we have

$$C\frac{dv_o(t)}{dt} + \frac{v_o(t) - V_s}{R} = 0$$

thus

$$\frac{dv_o(t)}{dt} = \frac{V_s}{CR} - \frac{v_o(t)}{CR} = 100 - 10v_0(t)$$

From Equation(5.4), the analytical solution is

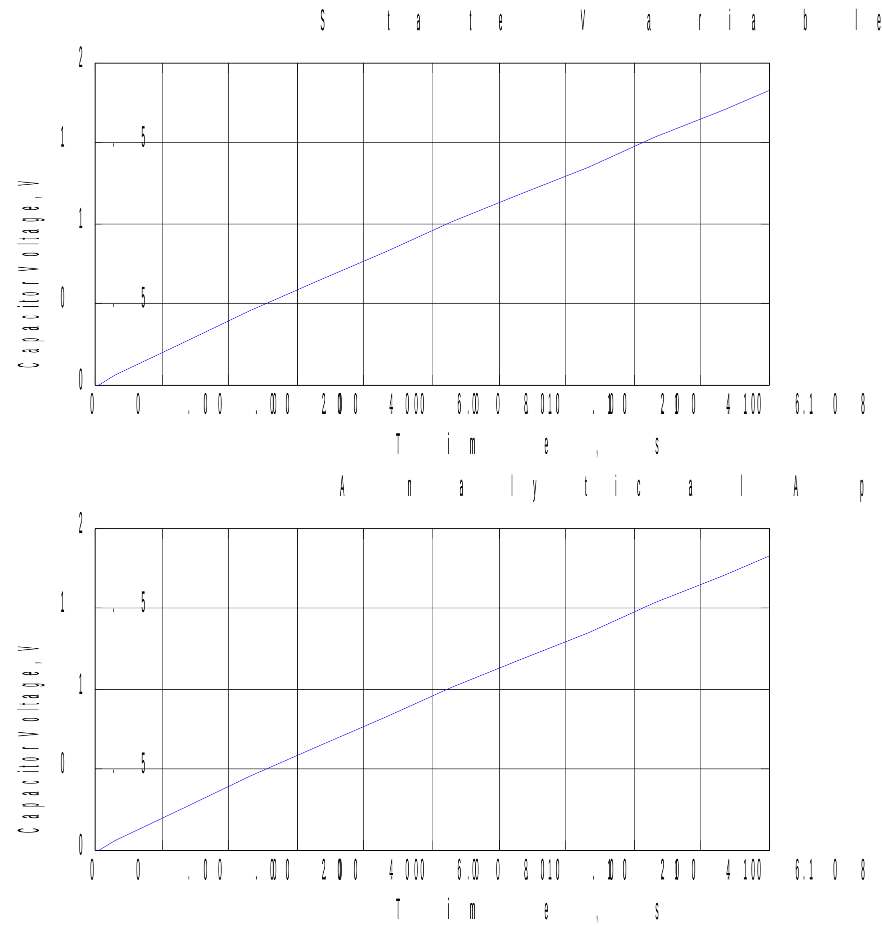
$$v_0(t) = 10\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)$$

Exemplo VII – Circuitos Elétricos

```
function dy = diff1(t,y)
dy = 100 - 10*y;
end
```

cv

```
% Condições Iniciais
t0 = 0;tf = 20e-3;xo = 0;
% Solução Numérica
[t, vo] = ode23('diff1',t0,tf,xo);
% Solução Analítica
vo_analy = 10*(1-exp(-10*t));
subplot(121) %Gráfico 01
plot(t,vo,'b');
title('Abordagem Numérica')
xlabel('Time, s'),
ylabel('Tensão Vc'),grid
subplot(122)%Gráfico 02
plot(t,vo_analy,'b');
title('Abordagem Analítica')
xlabel('Time, s'),
ylabel('Tensão Vc'),grid
```



Amanhã Novidades...Processamento de Sinais e Imagens!!!

Hoje prática!!!

Módulos		Conteúdos Teóricos
3º & 4º **23/09** **&** **24/09**	- Aplicações Partes I e II	- Parte I - Matemática; - Física; - Engenharias; - Parte II; - Estatística; - Processamento de Sinais; - Processamento de Imagens.

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